# Examination of direct-photon and pion production in proton–nucleon collisions

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## Abstract

We present a study of inclusive direct-photon and pion production in hadronic interactions, focusing on a comparison of the ratio of  $\gamma/\pi^0$  yields with expectations from next-to-leading order perturbative QCD (NLO pQCD). We also examine the impact of a phenomenological model involving  $k_T$  smearing (which approximates effects of additional soft-gluon emission) on absolute predictions for photon and pion production and their ratio.

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#### I. INTRODUCTION

Direct-photon production in hadronic collisions at high transverse momenta  $(p_T)$  has long been viewed as an ideal testing ground for the formalism of pQCD. A reliable theoretical description of the direct-photon process is of special importance because of its sensitivity to the gluon distribution in a proton through the quark–gluon scattering subprocess  $(gq \to \gamma q)$ . The gluon distribution, G(x), is relatively well constrained for x < 0.1 by data on deepinelastic lepton–nucleon scattering (DIS) and dilepton ("Drell-Yan") production, and by collider data on jet production at moderate x (0.1 – 0.25) [1], but is less constrained at larger x [2]. In principle, fixed-target direct-photon production can constrain G(x) at large x, and such data have therefore been incorporated in several modern global parton distribution function (PDF) analyses [1,3,4].

However, both the completeness of the theoretical NLO description of the direct-photon process, as well as the consistency of results from different experiments, have been subjects of intense debate [4–10]. The inclusive production of hadrons provides a further means of testing the predictions of the NLO pQCD formalism. Deviations have been observed between measured inclusive direct-photon and pion cross sections and NLO pQCD calculations. Examples of such discrepancies are shown in Figs. 1 and 2, where ratios of data to theory are displayed as a function of  $x_T = 2p_T/\sqrt{s}$  for  $p_T \gtrsim 3 \text{ GeV}/c$  for photon and pion data, respectively [11–22]. (Unless otherwise indicated, all NLO calculations [23–29] in this paper use a single scale of  $\mu = p_T/2$ , CTEQ4M PDFs [1], and BKK fragmentation functions for pions [30].) The uncertainties shown in the figures reflect mainly the statistical fluctuations in the measurements, except for some pioneering experiments from the ISR, which reported statistical errors combined with some of their systematic uncertainties.

Several of the older experiments quote large (of order 40%) uncertainties in their normalizations, making it difficult to judge the significance of the discrepancies observed in Figs. 1 and 2. Nevertheless, it is clear that both the photon and pion data show disagreement with theory, and the different data sets also disagree. This is not surprising, because, especially for direct-photon production, signals are often very small and difficult to extract, and backgrounds are quite large, especially at lower energies. Several experiments show better agreement with NLO pQCD than others, but the results do not provide great confidence in the theory nor in the quality of all the data. While it has been suggested that deviations from theory for both photons and pions can be ascribed to higher-order effects of initial-state soft-gluon radiation [6,7], it seems unlikely that theoretical developments alone will be able to accommodate the level of scatter observed in Figs. 1 and 2.

# II. $\gamma/\pi^0$ RATIO

Given the scatter of the data shown in Figs. 1 and 2, it may be instructive to consider measurements of the  $\gamma/\pi^0$  ratio in different experiments over a wide range of  $\sqrt{s}$ . Both experimental and theoretical uncertainties tend to cancel in such a ratio, and this ratio should also be less sensitive to incomplete treatment of gluon radiation. In addition, since  $\pi^0$  production constitutes the primary background to photon production, the  $\gamma/\pi^0$  ratio can serve as a measure of the difficulty of the experimental environment for making the direct-photon measurement.

The ratio of direct-photon to  $\pi^0$  cross sections for both data and NLO calculations (solid curves) are shown, as a function of  $x_T$ , in Figs. 3–7. Calculations for experiments using nuclear targets (E629, NA3, NA24, E706) were corrected approximately for nuclear dependence effects; these corrections were  $\lesssim 20\%$  for  $\pi^0$ 's and  $\lesssim 10\%$  for photons. The results from E704 [11], E629 [12], and NA3 [13], all at  $\sqrt{s} = 19.4$  GeV, are displayed in Fig. 3. For all three measurements, theory is high compared to data, somewhat less so for E629. Figure 4 shows the  $\gamma$  to  $\pi^0$  ratio for WA70 [14], NA24 [15], and UA6 [16] at  $\sqrt{s} \approx 23 - 24$  GeV. Just as for the comparisons at  $\sqrt{s} = 19.4$  GeV, theory is high relative to data, with the deviation between theory and data for WA70 and NA24 being greater than that for UA6.

For larger values of  $\sqrt{s}$ , the theoretically calculated value of the  $\gamma/\pi^0$  ratio agrees better with experiment. Figure 5 shows the data from R806 [18] at  $\sqrt{s} = 31$  GeV, and E706 [17] at  $\sqrt{s} = 31.6$  and 38.8 GeV. Theory provides a reasonable description of these measurements. For the results shown in Fig. 6, theory is now somewhat below the data for R806 [18] at  $\sqrt{s} = 45$  and 53 GeV, but agrees with the R807 data [31] at  $\sqrt{s} = 53$  GeV. Theory is also slightly low relative to data at  $\sqrt{s} = 63$  GeV for R806 [18,19], R807 [20], and R110 [21], as shown in Fig. 7. A compilation of these results, shown for simplicity without their uncertainties, is presented in Fig. 8. In this figure, the ratios of data to theory for the  $\gamma$  to  $\pi^0$  measurements shown in Figs. 3–7 have been fitted to a constant value at high- $p_T$ , and the results plotted as a function of  $\sqrt{s}$ . (Because the ratio depends somewhat on  $p_T$ , we excluded from the fit points at low  $p_T$ , until a  $\chi^2/d.o.f. = 1.5$  was reached for each fit.) The figure suggests an energy dependence in the ratio of data to theory for  $\gamma/\pi^0$  production. There is, however, an indication of substantial differences between the experiments at low  $\sqrt{s}$ , where the observed  $\gamma/\pi^0$  is smallest, which makes it difficult to quantify this trend. Recognizing the presence of these differences is especially important because thus far only the low energy photon experiments have been used in PDF fits to extract the gluon distribution.

#### III. PHENOMENOLOGICAL CORRECTIONS FOR SOFT GLUON EMISSION

In hadronic hard-scattering processes, there is generally a substantial amount of effective parton transverse momentum,  $k_T$ , in the initial state resulting from gluon emission [7,32]. The presence of  $k_T$  impacts the final state and has been observed in measurements of Drell-Yan, diphoton, and heavy quark production; the amount of  $k_T$  expected from NLO calculations is not sufficient to describe the data. The effective values of  $\langle k_T \rangle$ /parton for these processes vary from  $\approx 1 \text{ GeV}/c$  at fixed target energies to 3-4 GeV/c at the Tevatron Collider. The growth is approximately logarithmic with center-of-mass energy [7,32]. The size of the  $\langle k_T \rangle$  values, and their dependence on energy, argue against a purely "intrinsic" non-perturbative origin. Rather, the major part of this effect is generally attributed to soft gluon emission. While the importance of including gluon emission through the resummation formalism has long been recognized and such calculations have been available for some time for Drell-Yan [33], diphoton [34,35], and W/Z production [34], they have only recently been developed for inclusive direct-photon production [36–41].

In the absence of a more rigorous theoretical treatment of the impact of soft-gluon emission on high- $p_T$  inclusive production, a more intuitive, but successful, phenomenological approach has been utilized for describing the effects of soft-gluon radiation [7]. The soft-

gluon radiation was parametrized in terms of an effective  $\langle k_T \rangle$  that provided an additional transverse impulse to the outgoing partons. Because of the steeply falling cross section in  $p_T$ , such a  $\langle k_T \rangle$  can shift the production of final-state particles from lower to higher values of  $p_T$ , effectively enhancing the cross section. Using this intuitive picture, the effects of soft gluon emission are approximated through a convolution of the NLO cross section with a Gaussian  $k_T$  smearing function. The value of  $\langle k_T \rangle$  used for each kinematic regime is based on experimental data [7].

As described in Ref. [7], a leading order (LO) pQCD calculation [42] is used to generate correction factors (ratios of calculations for any given  $\langle k_T \rangle$  to the result for  $\langle k_T \rangle = 0$ ) for inclusive cross sections (Fig. 9). These  $p_T$ -dependent K-factors are then applied to the NLO pQCD calculations. This procedure involves a risk of double-counting since some of the  $k_T$ -enhancements may already be contained in the NLO calculation. However, the effects of such double-counting are expected to be small [7]. The relationship between this phenomenological  $k_T$ -smearing and the Collins–Soper–Sterman (CSS) resummation formalism [43] has been discussed in Ref. [44].

As illustrated in the upper part of Fig. 9, the K-factors for direct-photon production in E706 are large over the full range of  $p_T$ , and have  $p_T$ -dependent shapes. The lower part of Fig. 9 displays K-factors for  $\pi^0$  production, based on the same model. For the same values of  $\langle k_T \rangle$ , the K-factors in  $\pi^0$  production are somewhat smaller than in  $\gamma$  production. This is reasonable because  $\pi^0$  mesons originate from the fragmentation of partons, and therefore carry only a fraction of the  $k_T$  given to the jet. Because the momentum fraction of the jet carried by a  $\pi^0$  tends to increase with increasing  $p_T$  of the  $\pi^0$  (sometimes referred to as trigger-bias), the  $k_T$  corrections in the case of  $\pi^0$  production are smaller and tend to have a weaker  $p_T$ -dependence than the corrections for direct photons.

In this model, the size of the correction depends sensitively on the value used for  $\langle k_T \rangle$ . Changes of 200 MeV/c produce substantial differences in the size of the correction. Values of  $\langle k_T \rangle$  are only accurate to about this range [7], and so, even within the context of this model, it is difficult to obtain the kind of precision needed for extracting global parton distributions. In addition, there are different ways to implement the  $k_T$ -corrections [4], which can produce quantitative differences in the magnitude and shape of the  $k_T$  correction factors.

We have investigated the kind of K-factors that would be expected for direct-photon production from parton-showering models [45,46]. These programs do not provide sufficient smearing at fixed-target energies because shower development is constrained by cut-off parameters that ensure the perturbative nature of the process. Consequently, these calculations allow additional input  $k_T$  for Gaussian smearing, and are often used that way in comparisons to data (default values are  $\langle k_T \rangle = 0$  for HERWIG and  $\langle k_T \rangle = 0.9 \text{ GeV}/c$  for PYTHIA). Using default settings for other program parameters and an input  $\langle k_T \rangle$  of 1.2 GeV/c (to compare to our previous results [7]) for the smearing, relative to these same settings with  $k_T = 0$  in these generators, we obtain the K-factors shown in Fig. 10. These corrections are different than those found using the LO pQCD prescription [42]. These differences must be kept in mind when making comparisons to data. Applying the K-factors in Fig. 10 to NLO pQCD results in the comparison to data from E706 shown in Fig. 11.

The dashed curves in Figs. 3–7 describe the predicted ratios of  $\gamma$  and  $\pi^0$  cross sections using the previous  $k_T$  corrections [7]. The impact of  $k_T$  corrections upon  $\gamma/\pi^0$  is generally minimal, indicating that the trend observed in Fig. 8 cannot be easily understood on the

basis of corrections for  $k_T$  alone.

#### IV. RECENT PROGRESS IN THEORY

Resummed pQCD calculations for single direct-photon production are currently under development [36–41]. Substantial corrections to fixed-order QCD calculations are expected from soft-gluon emission, especially in regions of phase space where gluon emission is restricted kinematically. Such restrictions occur, for example, in the production of Drell-Yan or diphoton pairs at low  $p_T$ , where effects of soft-gluon emission have to be resummed in order to obtain adequate description of cross sections [33]. In addition, at high x, there is a similar suppression of gluon radiation due to the rapidly falling parton distributions. A complete description of the cross section in this region requires the resummation of "threshold" terms. For inclusive "single-arm" observables, there is no restriction in phase space for soft gluon radiation, and therefore no enhancement to the cross section is expected from traditional resummation calculations except at high x [47]. Two recent independent threshold-resummed pQCD calculations for direct photons [36,37] exhibit far less dependence on QCD scales than found in NLO theory. These calculations agree with the NLO prediction for the scale  $\mu \approx p_T/2$  at low  $p_T$  (without inclusion of explicit  $k_T$  or recoil effects), and show an enhancement in cross section at high  $p_T$  due to the threshold terms.

A method for simultaneous treatment of recoil and threshold corrections in inclusive single-photon cross sections is being developed [41] within the formalism of collinear factorization. This approach accounts explicitly for the recoil from soft radiation in the hard-scattering subprocess, and conserves both energy and transverse momentum for the resummed radiation. The possibility of substantial enhancements from higher-order perturbative and power-law nonperturbative corrections relative to NLO are indicated at both moderate and high  $p_T$  for fixed-target energies, similar to the enhancements obtained with the simple  $k_T$ -smearing model discussed above.

Figure 12 displays the results of an example calculation based on the latest approach involving both threshold and recoil resummation for comparison with direct-photon measurements from E706 [41]. As shown in Fig. 12, this theoretical result is substantially higher than the prediction from NLO pQCD, higher than the theory using just threshold resummation, and closer to the phenomenological  $k_T$ -smearing model used in Fig. 11.

#### V. CONCLUSIONS

We have examined experimental information on the production of direct photons and pions at large  $p_T$  and reviewed how LO pQCD has been used to estimate the impact of  $k_T$  on their inclusive production. Simple phenomenological models can improve agreement between pQCD calculations and much of the inclusive data over a wide range of  $\sqrt{s}$  [7]. While the deficiencies of such models are clear, and have been discussed in recent literature, the emerging formalism for the full (threshold and recoil) resummation of inclusive direct-photon cross sections [41] appears to support some of the understanding of soft multiple gluon emission that has been gained using the approximate  $k_T$  tools.

Despite the apparent inconsistencies between different direct-photon experiments [7–9], we found it instructive to consider results on the measured  $\gamma/\pi^0$  ratios since various experimental and theoretical uncertainties tend to cancel in such ratios. We find that the ratios taken from theory agree reasonably with those computed from data for  $\sqrt{s} \gtrsim 30$  GeV. However, there appears to be a systematic trend with energy that is unexplained, and differences between experiments that are particularly significant at lower energy.

While there is still no resummation calculation for inclusive pion production, the trend of recent developments in direct-photon processes has led to an increased appreciation of the importance of the effect of multiple gluon emission, and to the emergence of tools for incorporating these effects. These latest theoretical developments encourage optimism that the long-standing difficulties in developing an adequate description of these processes can eventually be resolved, making possible a global re-examination of parton distributions with an emphasis on the determination of the gluon distribution from the direct-photon data [48].

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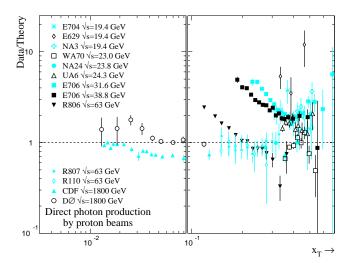


FIG. 1. Comparison between proton-induced direct-photon data and NLO pQCD calculations as a function of photon  $x_T$ .

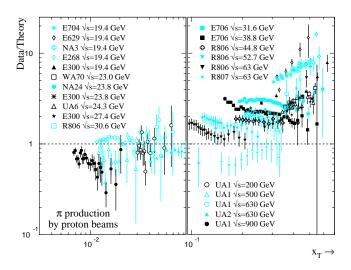


FIG. 2. Comparison between proton-induced pion data and NLO pQCD calculations for experiments spanning  $\sqrt{s} = 19.4$  to 900 GeV as a function of  $x_T$ .

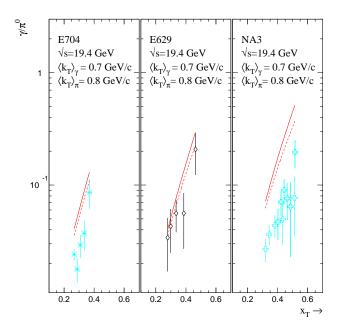


FIG. 3. Comparison of  $\gamma/\pi^0$  rates as a function of  $x_T$  for E704, E629, and NA3 at  $\sqrt{s} = 19.4$  GeV. Overlayed are the results from NLO pQCD (solid) and  $k_T$ -enhanced calculations (dashed). Values of  $\langle k_T \rangle$  used for the  $k_T$ -enhanced calculations are given in the legend.

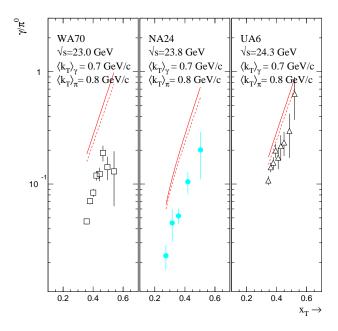


FIG. 4. Comparison of  $\gamma/\pi^0$  rates as a function of  $x_T$  for WA70, NA24, and UA6 at  $\sqrt{s} \approx 23-24$  GeV. Overlayed are the results from NLO pQCD (solid) and  $k_T$ -enhanced calculations (dashed). Values of  $\langle k_T \rangle$  used for the  $k_T$ -enhanced calculations are given in the legend.

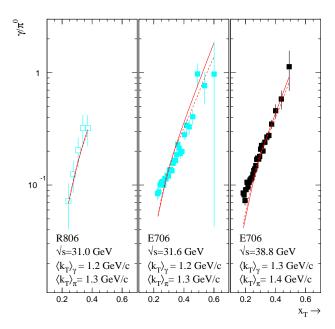


FIG. 5. Comparison of  $\gamma/\pi^0$  rates as a function of  $x_T$  for R806 at  $\sqrt{s}=31$  GeV and E706 at  $\sqrt{s}=31.6$  and 38.8 GeV. Overlayed are the results from NLO pQCD (solid) and  $k_T$ -enhanced calculations (dashed). Values of  $\langle k_T \rangle$  used for the  $k_T$ -enhanced calculations are given in the legend.

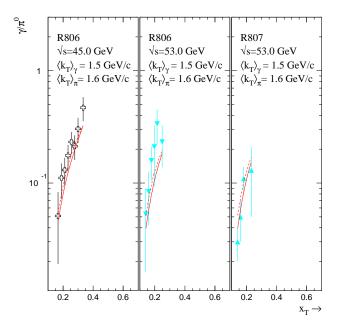


FIG. 6. Comparison of  $\gamma/\pi^0$  rates as a function of  $x_T$  for R806 at  $\sqrt{s}=45$  and 53 GeV and R807 at  $\sqrt{s}=53$  GeV. Overlayed are the results from NLO pQCD (solid) and  $k_T$ -enhanced calculations (dashed). Values of  $\langle k_T \rangle$  used for the  $k_T$ -enhanced calculations are given in the legend.

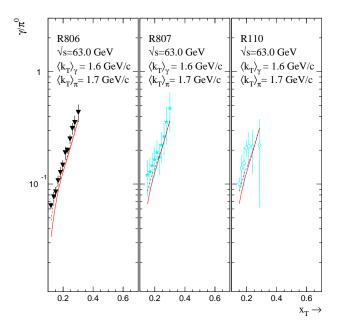


FIG. 7. Comparison of  $\gamma/\pi^0$  rates as a function of  $x_T$  for R806, R807, and R110 at  $\sqrt{s}=63$  GeV. Overlayed are the results from NLO pQCD (solid) and  $k_T$ -enhanced calculations (dashed). Values of  $\langle k_T \rangle$  used for the  $k_T$ -enhanced calculations are given in the legend.

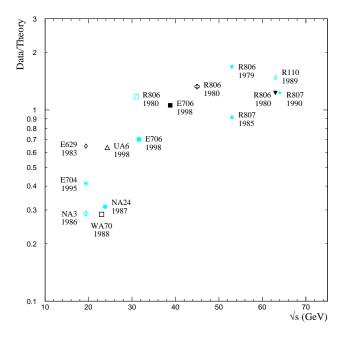


FIG. 8. The ratio of data to theory for the  $\gamma/\pi^0$  comparisons presented as a function of  $\sqrt{s}$  for the direct-photon experiments considered in this paper. The values represent fits to the ratio of data to NLO pQCD theory, without  $k_T$ -enhancement (see text).

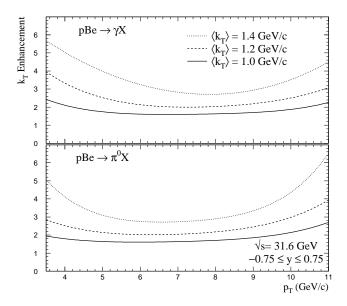


FIG. 9. The dependence of the  $k_T$ -enhancement on  $p_T$ ,  $K(p_T)$ , for variations in  $\langle k_T \rangle$  for the E706 pBe data at  $\sqrt{s} = 31.6$  GeV.

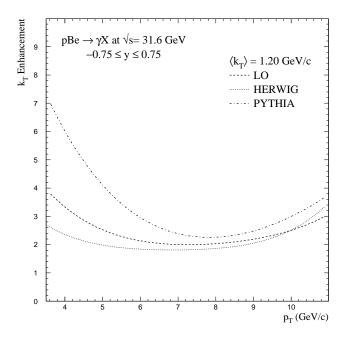


FIG. 10. Comparison between the  $k_T$  enhancements,  $K(p_T)$ , from the LO pQCD calculation (dashed), HERWIG (dotted), and PYTHIA (dashed-dotted).

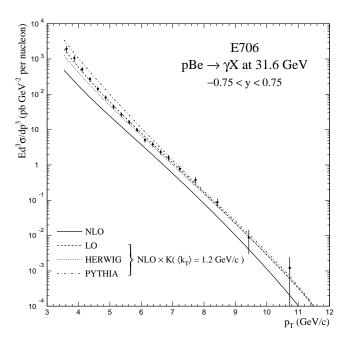


FIG. 11. Comparison between the E706 direct-photon data at  $\sqrt{s} = 31.6$  GeV and the NLO pQCD calculation(solid), and the NLO theory enhanced by K-factors associated with the LO calculation (dashed), HERWIG (dotted), and PYTHIA (dashed-dotted).

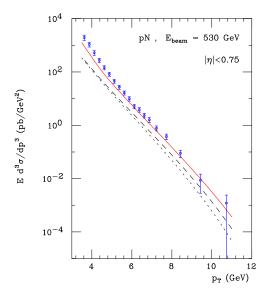


FIG. 12. Direct-photon cross section for the E706 data at  $\sqrt{s} = 31.6$  GeV compared to recent QCD calculations. The dotted line represents the full NLO calculation [28], while the dashed and solid lines, respectively, incorporate purely threshold resummation [36] and joint threshold and recoil resummation [41].